This very interesting study is marred by a problematic analysis of what I take to be the heart of the matter, the comparison of what Einstein thought about the astronomical situation in cosmology in the early 1930s with what he could have known if he had cared to take the trouble. To explain the problem I review the textbook analysis, which is tedious but aids clarity. This was not in the textbooks in 1931, of course, but it is not that difficult; Einstein could have handled it.

In notation more familiar to me (with the velocity of light set to unity) equation (2a) in the paper is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{1}{a^2 R^2}. \tag{1}$$

We're assuming pressure may be neglected, so the mass density varies as $\rho(t) \propto a(t)^{-3}$. Then the parametric solution to (1) is

$$a = A(1 - \cos\eta), \qquad t = B(\eta - \sin\eta), \tag{2}$$

where A and B are constants. The results of substituting (2) into (1) are

$$B = AR, \qquad \frac{A^3}{B^2} = \frac{4}{3}\pi G\rho a^3. \tag{3}$$

With $M = 2\pi^2\rho a^3 R^3$ defined as the product of the mass density $\rho(t)$ with the volume of the closed universe at fixed world time t, we have

$$B = \frac{2}{3\pi}GM, \qquad t_{2\pi} = 2\pi B = 4GM/3, \tag{4}$$

where $t_{2\pi}$ is the time from start of expansion to end of contraction. This is the authors' second equation on page 17.

But consider that astronomers — and Einstein — could compare the cosmological model to estimates of the present mass density ρ_o, the expansion time t_o, and Hubble's constant $H_o = \dot{a}/a$ at t_o, for which

$$H_o^2 = \frac{4}{3}\pi G\rho_o(1 + \cos\eta_o), \qquad H_o B = \frac{\sin\eta_o}{(1 - \cos\eta_o)^2}, \qquad H_o t_o = \frac{\sin\eta_o(\eta_o - \sin\eta_o)}{(1 - \cos\eta_o)^2}. \tag{5}$$

Given astronomers' estimates of Hubble's constant H_o and the present mass density ρ_o, the first of these equations gives the present value η_o of η, the last then gives the expansion time to the present, and the second equation gives $t_{2\pi} = 2\pi B$. The limit $\eta_o \to 0$ is the Einstein-de Sitter case,

$$H_o^2 = \frac{8}{3}\pi G\rho_o, \qquad H_o t_o = 2/3. \tag{6}$$

In terms of H_o and ρ_o the total time $t_{2\pi}$ from the start of expansion from $P = 0$ to contraction back to $P = 0$, as η increases from 0 to 2π, is

$$t_{2\pi} = 2\pi B = \frac{2\pi}{H_o} \frac{\sin \eta_o}{(1 - \cos \eta_o)^2}. \tag{7}$$

In the Einstein-de Sitter limit, $\eta_o \to 0$, $t_{2\pi} \to \infty$. That is, the "timespan of the cycle" mentioned on page 17 can be arbitrarily large. If H_o is given and one is willing to assume an arbitrarily large present mass density ρ_o, so that η_o might approache π from below, then we see from equation (7) that $t_{2\pi}$ can be arbitrarily small. That would make t_o arbitrarily small too, but since there was a timescale problem even for the Einstein de Sitter case maybe a short t_o is no more unlikely.

The point is that the parameter M in the authors' expression for $t_{2\pi}$ is not an observable available to astronomers in the 1930s, or to Einstein. It is misleading to quote a value for M on page 17 that has no meaning for astronomers in the 1930s.

The authors remark that Einstein's value for the Hubble time H_o^{-1} is an order of magnitude low. They might have added that in a closed geometry with $\Lambda = 0$ the expansion time must be less than $2/(3H_o)$, which Einstein could easily have seen if he had tried. That is, his value $t_o \sim 10^{10}$ years is absurd. But also out of place are the authors' remarks on pages 16 and 17 about "Einstein's short timeframe of the model", because his t_o, though wrong, is larger than the radioactive decay ages under discussion then (or now). If Einstein's elapsed time had been accurate there would have been no timescale problem.

I draw the authors' attention to several other considerations.

p 3-10: The word "predict" is unfortunate, for GR allows collapsing as well as expanding solutions.

p 9+5: Did Einstein (1917) "discover that" GR "predicted a universe of time-varying radius"? In the paragraph after equation (13) in Einstein (1917) I see the comment that his stress-energy tensor, which assumes negligible pressure, cannot satisfy his original field equation. Of course, that is because he takes it that spacetime is static. I see no mention of cosmic evolution in Einstein (1917).

p 12 second paragraph: since P is given to a first significant figure the authors might as well do the same for the mass density, $\rho = 4 \times 10^{28}$. And why mix units, meters and centimeters, here and on p 15?

p 20-6: What is meant by the "nature of the gravitational potential"?

p 22+4: Cosmic expansion can be understood without GR: consider a Newtonian description of an expanding cloud of galaxies.

p 22-6: Friedman could not be expected to have considered the age issue: he didn't know about the astronomical observations of galaxy redshifts.

Finally, although choices of style should largely be left to the authors, I suggest the authors consider that this paper could be considerably shorter and thus more readable without removing any content. I offer one example, that the abstract and Introduction are almost duplicates.