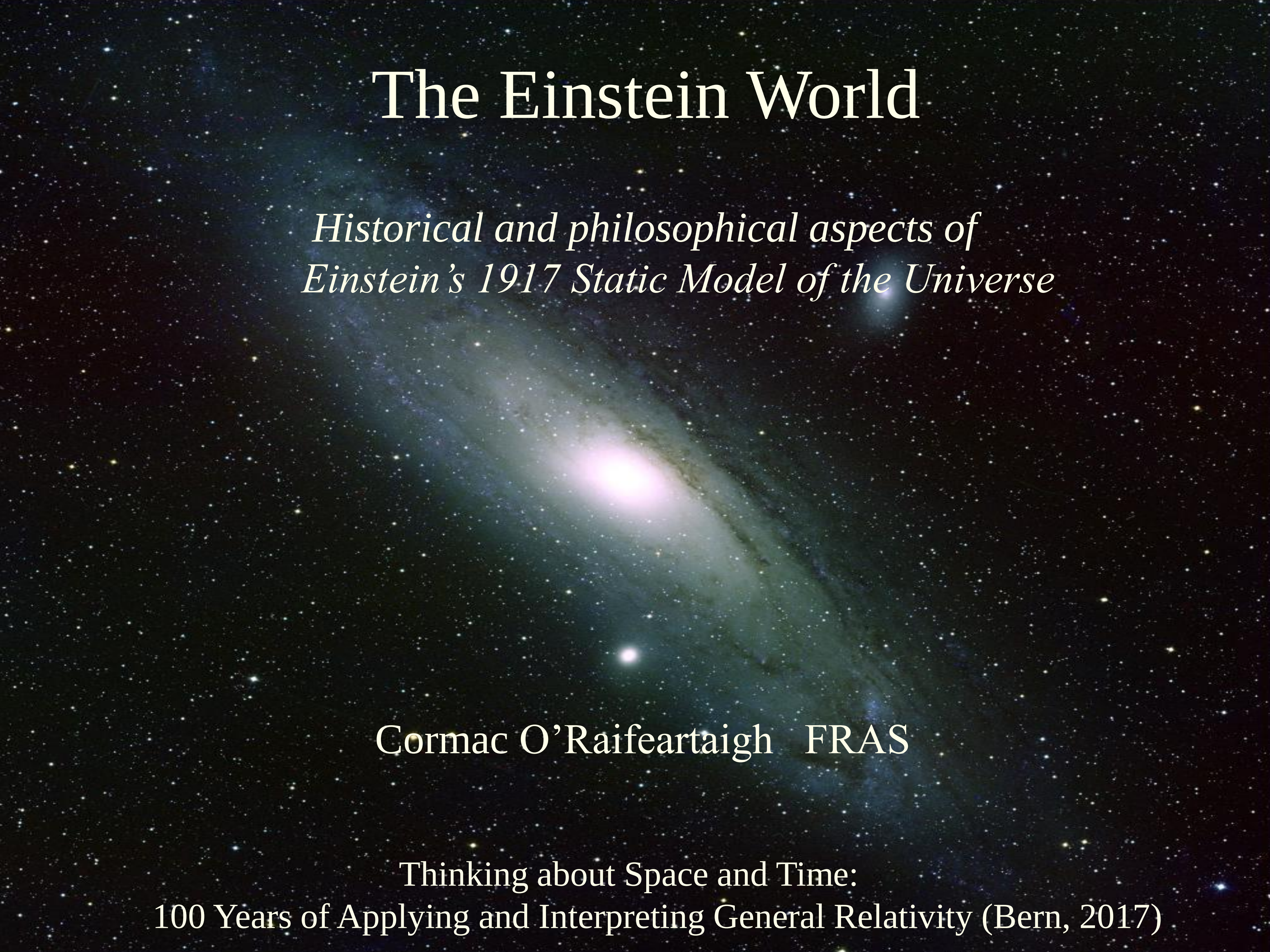


The Einstein World

Historical and philosophical aspects of Einstein's 1917 Static Model of the Universe

Cormac O'Raifeartaigh FRAS

Thinking about Space and Time:
100 Years of Applying and Interpreting General Relativity (Bern, 2017)

Overview

- **Historical remarks**

  Biographical context (1915-1917)

  Scientific context: from GR to cosmology

- **Einstein's 1917 model of the cosmos**

  Basic assumptions: basic principles

  A guided tour

  Theoretical, empirical and philosophical issues

- **Einstein and alternate cosmologies**

  Einstein vs de Sitter, Friedman, Lemaître

  Einstein's expanding models

Conclusions


The European Physical Journal H
August 2017, Volume 42, Issue 3, pp 431–474

Einstein's 1917 static model of the universe: a centennial review

Authors   Authors and affiliations

Cormac O'Raifeartaigh, Michael O'Keeffe, Werner Nahm, Simon Mitton

I Historical remarks

- **Appointed to Berlin Chair**

  Arrives April 1914

  Family leave Berlin, June 1914

- **World War I (1914-18)**

  Living alone, food shortages

  Dietary problems, illness

- **Second 'miraculous' period**

  Covariant field equations (1915)

  Exposition, solutions and predictions (1916)

  <u>*First relativistic model of the cosmos (1917)*</u>

  Papers on gravitational waves

  Papers on the quantum theory of radiation

  Papers on unified field theory

Einstein in Berlin (1916)

Scientific context

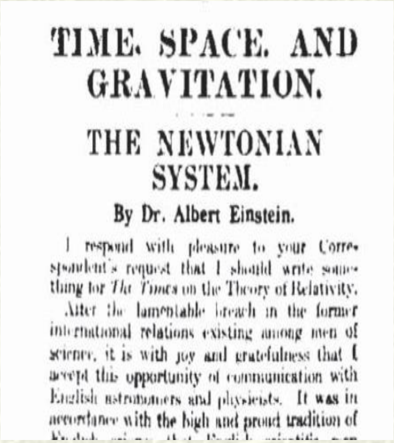
<u>*Recall GR = 'principle-led' theory*</u>

- **The general principle of relativity (1907-)**
  Relativity and accelerated motion

- **The principle of equivalence**
  Equivalence of gravity and acceleration

- **The principle of Mach**
  Relativity of inertia
  Structure of space determined by matter
  No space without matter

<u>*Some cosmological considerations 'built in' to GR*</u>

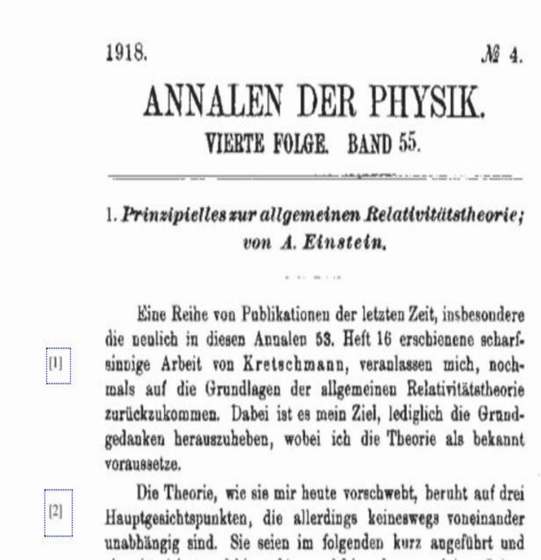


TIME. SPACE. AND GRAVITATION.

THE NEWTONIAN SYSTEM.

By Dr. Albert Einstein.

I respond with pleasure to your Correspondent's request that I should write something for *The Times* on the Theory of Relativity. After the lamentable breach in the former international relations existing among men of science, it is with joy and gratefulness that I accept this opportunity of communication with English astronomers and physicists. It was in accordance with the high and proud tradition of



1918. № 4.

ANNALEN DER PHYSIK.

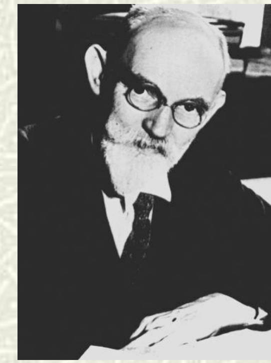
VIERTE FOLGE. BAND 55.

1. ***Prinzipielles zur allgemeinen Relativitätstheorie; von A. Einstein.***

[1] Eine Reihe von Publikationen der letzten Zeit, insbesondere die neulich in diesen Annalen 53. Heft 16 erschienene scharfsinnige Arbeit von Kretschmann, veranlassen mich, nochmals auf die Grundlagen der allgemeinen Relativitätstheorie zurückzukommen. Dabei ist es mein Ziel, lediglich die Grundgedanken herauszuheben, wobei ich die Theorie als bekannt voraussetze.

[2] Die Theorie, wie sie mir heute vorschwebt, beruht auf drei Hauptgesichtspunkten, die allerdings keineswegs voneinander unabhängig sind. Sie seien im folgenden kurz angeführt und

Relativistic cosmology (1915-17)



- **A natural progression**

  Ultimate test for any theory of gravitation

  Ultimate test for Mach's principle

- **<u>Assumption 1</u>: static universe**

  Observation, experience (QA)

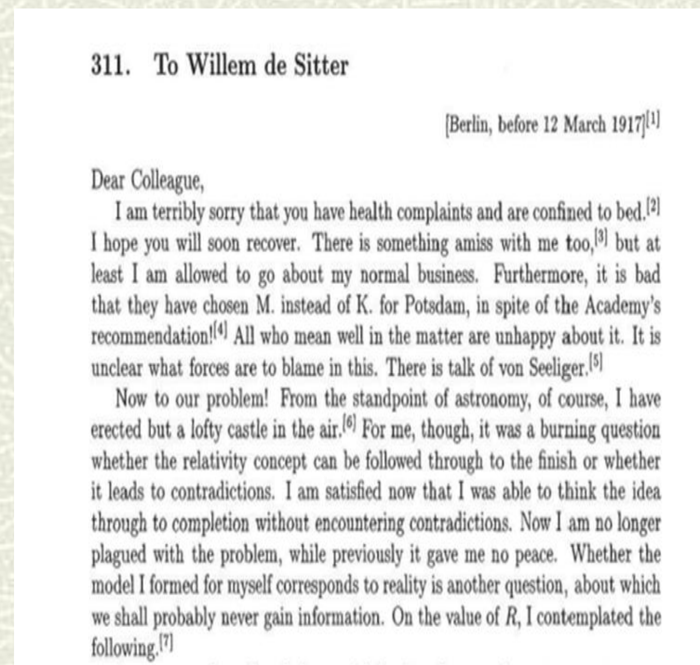
- **<u>Assumption 2</u>: uniform distribution of matter**

  Simplicity (Copernican principle?)

- **<u>Assumption 3/Principle:</u> Mach's principle**

  No space without matter

<u>Boundary conditions at infinity?</u>



311. To Willem de Sitter

[Berlin, before 12 March 1917][1]

Dear Colleague,

I am terribly sorry that you have health complaints and are confined to bed.[2] I hope you will soon recover. There is something amiss with me too,[3] but at least I am allowed to go about my normal business. Furthermore, it is bad that they have chosen M. instead of K. for Potsdam, in spite of the Academy's recommendation![4] All who mean well in the matter are unhappy about it. It is unclear what forces are to blame in this. There is talk of von Seeliger.[5]

Now to our problem! From the standpoint of astronomy, of course, I have erected but a lofty castle in the air.[6] For me, though, it was a burning question whether the relativity concept can be followed through to the finish or whether it leads to contradictions. I am satisfied now that I was able to think the idea through to completion without encountering contradictions. Now I am no longer plagued with the problem, while previously it gave me no peace. Whether the model I formed for myself corresponds to reality is another question, about which we shall probably never gain information. On the value of R, I contemplated the following.[7]

The problem of boundary conditions

- **Flat space-time at infinity?**
  Privileged reference frame
  Contrary to Mach's principle

- **Degenerate $g_{\mu\nu}$ at infinity?**
  Einstein in Leyden (Autumn 1916)
  Difficult to reconcile with observation (de Sitter)

- **Einstein's ingenious solution**
  Remove the boundaries! (November, 1916)
  A universe of closed spatial geometry

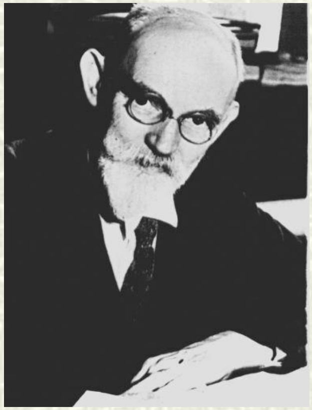
"I have perpetrated something which exposes me ..
to the danger of being committed to a madhouse"

On Einstein's Theory of Gravitation, and its Astronomical Consequences. By W. de Sitter, Assoc. R.A.S. Second Paper.*

Contents of Second Paper.

20. Field of n moving bodies: the differential equations.
21. First method of integration.
22. Second method of integration: comparison of the two methods.
23. Problem of n bodies: equations of motion. Planetary theories.
24. Lunar theory: equations of motion.
25. Further development, perturbing forces.
26. Motion of lunar perigee and node: comparison with observations.
27. Field of moving sun and motion of infinitesimal planet in it, compared with field of sun at rest.
28. Gravitational field of the system of the fixed stars. Effect on

$$\left\{\begin{matrix} 0 & 0 & 0 & \infty \\ 0 & 0 & 0 & \infty \\ 0 & 0 & 0 & \infty \\ \infty & \infty & \infty & \infty^2 \end{matrix}\right.$$



II A guided tour of the paper

Doc. 43

Cosmological Considerations in the General Theory of Relativity

[1]

This translation by W. Perrett and G. B. Jeffery is reprinted from H. A. Lorentz et al., *The Principle of Relativity* (Dover, 1952), pp. 175–188.

It is well known that Poisson's equation

$$\nabla^2\phi = 4\pi K\rho \quad . \quad . \quad . \quad . \quad (1)$$

in combination with the equations of motion of a material point is not as yet a perfect substitute for Newton's theory of action at a distance. There is still to be taken into account the condition that at spatial infinity the potential ϕ tends toward a fixed limiting value. There is an analogous state of things in the theory of gravitation in general relativity. Here, too, we must supplement the differential equations by limiting conditions at spatial infinity, if we really have to regard the universe as being of infinite spatial extent. [2]

In my treatment of the planetary problem I chose these limiting conditions in the form of the following assumption: it is possible to select a system of reference so that at spatial infinity all the gravitational potentials $g_{\mu\nu}$ become constant. [3] But it is by no means evident *a priori* that we may lay down the same limiting conditions when we wish to take larger portions of the physical universe into consideration. In the following pages the reflexions will be given which, up to the present, I have made on this fundamentally important question.

§ 1. The Newtonian Theory

It is well known that Newton's limiting condition of the constant limit for ϕ at spatial infinity leads to the view that the density of matter becomes zero at infinity. For we imagine that there may be a place in universal space round about which the gravitational field of matter, viewed on a large scale, possesses spherical symmetry. It then follows from Poisson's equation that, in order that ϕ may tend to a

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential ϕ einem festen Grenzwerte zustrebt. Analog verhält es sich bei der Gravitationstheorie der allgemeinen Relativität; auch hier müssen zu den Differentialgleichungen Grenzbedingungen hinzutreten für das räumlich Unendliche, falls man die Welt wirklich als räumlich unendlich ausgedehnt anzusehen hat.

Bei der Behandlung des Planetenproblems habe ich diese Grenzbedingungen in Gestalt folgender Annahme gewählt: Es ist möglich, ein Bezugssystem so zu wählen, daß sämtliche Gravitationspotentiale $g_{\mu\nu}$ im räumlich Unendlichen konstant werden. Es ist aber a priori durchaus nicht evident, daß man dieselben Grenzbedingungen ansetzen darf, wenn man größere Partien der Körperwelt ins Auge fassen will. Im folgenden sollen die Überlegungen angegeben werden, welche ich bisher über diese prinzipiell wichtige Frage angestellt habe.

§ 1. Die Newtonsche Theorie.

Es ist wohlbekannt, daß die Newtonsche Grenzbedingung des konstanten Limes für ϕ im räumlich Unendlichen zu der Auffassung hinführt, daß die Dichte der Materie im Unendlichen zu null wird. Wir denken uns nämlich, es lasse sich ein Ort im Weltraum finden, um den herum das Gravitationsfeld der Materie, im großen betrachtet, Kugelsymmetrie besitzt (Mittelpunkt). Dann folgt aus der Poissonschen Gleichung, daß die mittlere Dichte ρ rascher als $\frac{1}{r^2}$ mit wachsender Entfernung r vom Mittelpunkt zu null herabsinken muß, damit ϕ im

Structure of Einstein's 1917 paper

§ 1. The Newtonian Theory

§ 2. The Boundary Conditions According to the General Theory of Relativity

§ 3. The Spatially Finite Universe with a Uniform Distribution of Matter

§ 4. On an Additional Term for the Field Equations of Gravitation

§ 5. Calculation and Result

$$G_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

1. The Newtonian theory

- **Divergence of gravitational force**
  Assuming non-zero, uniform density of matter
  Well-known paradox (Bentley-Newton)

$$\nabla^2\phi = 4\pi G\rho \quad \textbf{(P1)}$$

$$\phi = G\int \frac{\rho\,(r)}{r}\,dV$$

- **Einstein's formulation of problem**
  Mean density must decrease more rapidly than $1/r^2$
  for constant gravitational potential at infinity: island solution

- **Stability paradox**
  Island of matter unstable statistically
  Evaporation argument $\rho_\infty = 0 \rightarrow \rho_c = 0$

- **Solution: modify Poisson's equation**
  Finite solution for potential
  "A foil for what is to follow"

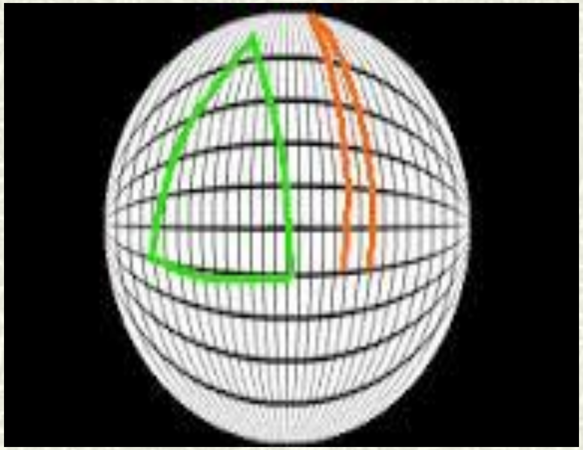
$$\nabla^2\phi - \lambda\phi = 4\pi G\rho \quad \textbf{(P2)}$$

$$\phi = -\frac{4\pi}{\lambda}G\rho$$

Independent of modifications by Seeliger, Neumann

3. The spatially closed universe
4. An additional term in the GFE

- **Assume stasis** *(the Known Universe)*
  Assume non-zero uniform density of matter



- **Introduce closed spatial curvature**
  To conform with Mach's principle
  Solves problem of $g_{\mu\nu}$

$$G_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

- **<u>Null result</u>**
  "GFE not satisfied with these values of $g_{\mu\nu}$"

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

- **Introduce new term in GFE***
  Additional term needed in field equations

The need for a cosmological constant

From 3(a), in accordance with (1a) one calculates for the $R_{\mu\nu}$ $(x_1 = x_2 = x_3 = 0)$ the values

$$\begin{matrix} -\frac{2}{P^2} & 0 & 0 & 0 \\ 0 & -\frac{2}{P^2} & 0 & 0 \\ 0 & 0 & -\frac{2}{P^2} & 0 \\ 0 & 0 & 0 & 0, \end{matrix}$$

for $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, the values

$$\begin{matrix} \frac{1}{P^2} & 0 & 0 & 0 \\ 0 & \frac{1}{P^2} & 0 & 0 \\ 0 & 0 & \frac{1}{P^2} & 0 \\ 0 & 0 & 0 & -\frac{3c^2}{P^2}, \end{matrix}$$

while for $-\kappa \boldsymbol{T}$ one obtains the values

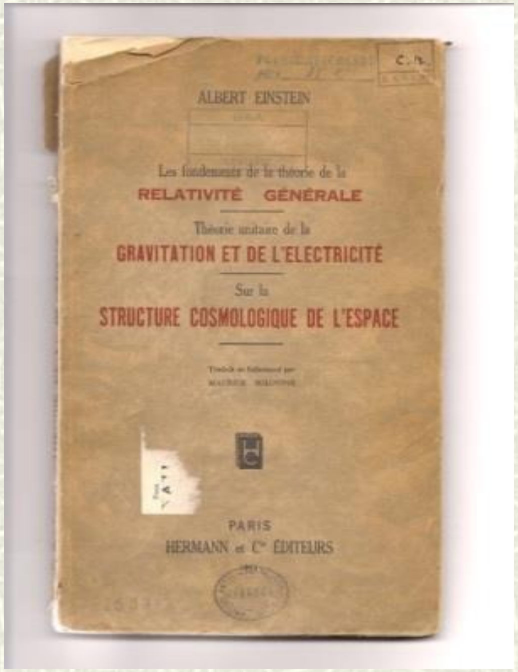
$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\kappa\rho c^2 \end{matrix}$$

Thus from (1) the two contradictory equations are obtained

$$\left. \begin{aligned} \frac{1}{P^2} &= 0 \\ \frac{3c^2}{P^2} &= \kappa\rho c^2 \end{aligned} \right\} (4)$$

$$\boldsymbol{G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa\, T_{\mu\nu}}$$

$$ds^2 = \frac{dx_1^2 + dx_2^2 + dx_3^2}{\left(1 + \frac{r^2}{(2P)^2}\right)^2} - c^2 dt^2$$

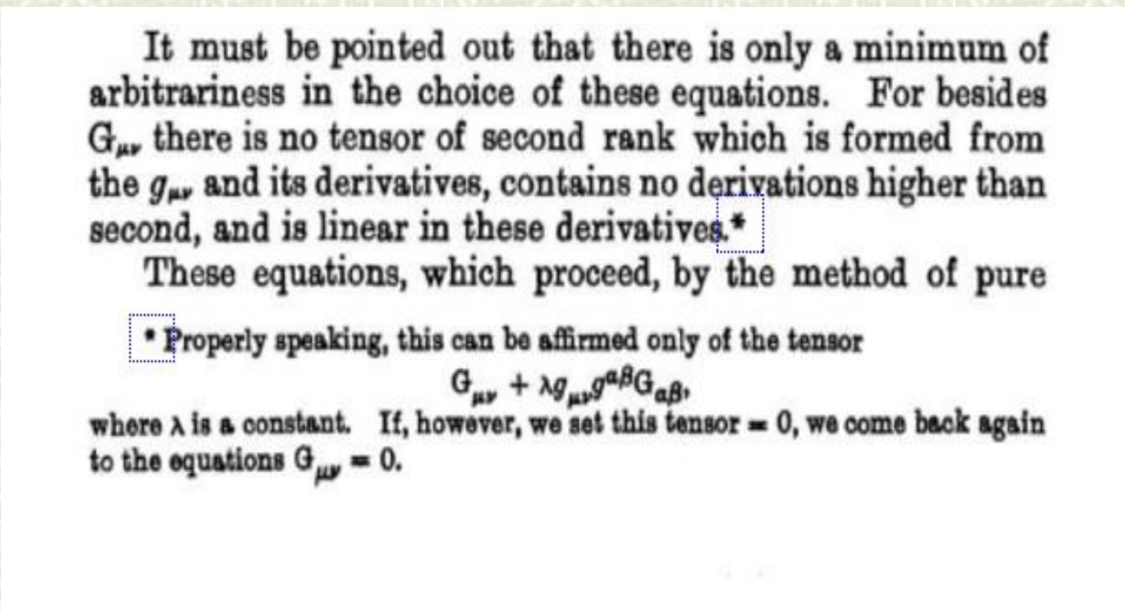


Einstein 1933

λ term needed for (static) solution

A precursor for the cosmological constant

- λ introduced in 1916? *Ann. Physik*. **49:** 769-822
- The field equations in the absence of matter



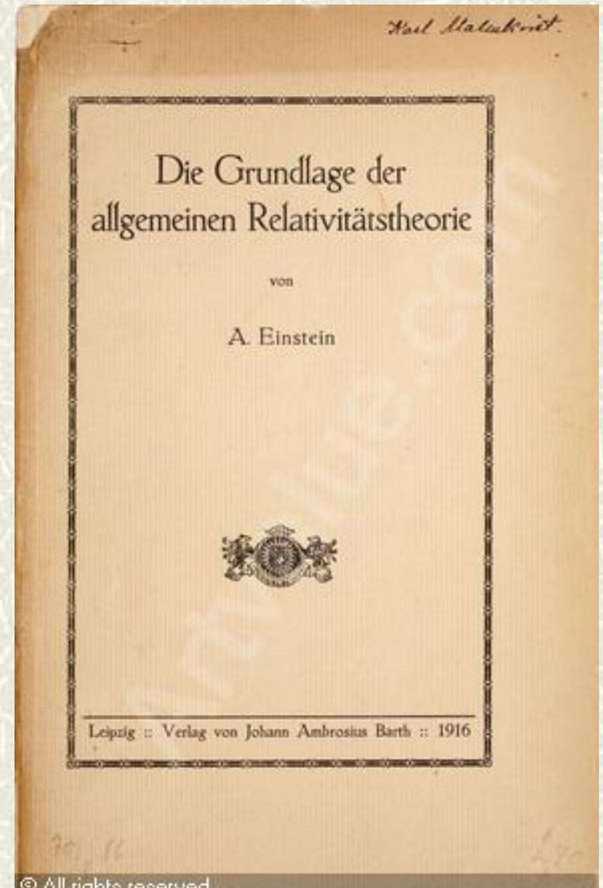
It must be pointed out that there is only a minimum of arbitrariness in the choice of these equations. For besides $G_{\mu\nu}$ there is no tensor of second rank which is formed from the $g_{\mu\nu}$ and its derivatives, contains no derivations higher than second, and is linear in these derivatives.*

These equations, which proceed, by the method of pure

* Properly speaking, this can be affirmed only of the tensor

$$G_{\mu\nu} + \lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta},$$

where λ is a constant. If, however, we set this tensor = 0, we come back again to the equations $G_{\mu\nu} = 0$.



- *Prepared the way for $\lambda g_{\mu\nu}$ in 1917*

5. Calculation and result

- **Calculation and result**

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

Thus the newly introduced universal constant λ defines both the mean density of distribution ρ which can remain in equilibrium and also the radius R and the volume $2\pi^2R^3$ of spherical space. The total mass M of the universe, accord-

curvature of space is variable in time and place, according to the distribution of matter, but we may roughly approximate to it by means of a spherical space. At any rate, this view is logically consistent, and from the standpoint of the general theory of relativity lies nearest at hand; whether, from the standpoint of present astronomical knowledge, it is tenable, will not here be discussed. In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified by our actual knowledge of gravitation. It is to be emphasized, however, that a positive curvature of space is given by our results, even if the supplementary term is not introduced. That term is necessary only for the purpose of making possible a quasi-static distribution of matter, as required by the fact of the small velocities of the stars.

- **Caveats**

Consistent model without reference to astronomy
Extension of GFE required
Necessitated by assumption of stasis

On the cosmological constant (i)

- **Introduced in analogy with Newtonian cosmology**
  Full section on Newtonian gravity (Einstein 1917)
  Indefinite potential at infinity? Problem of stability

$$\nabla^2\phi = 4\pi G\rho \quad \text{(P1)}$$

$$\nabla^2\phi - \lambda\phi = 4\pi G\rho \quad \text{(P2)}$$

- **Modifying Newtonian gravity**
  Extra term in Poisson's equation

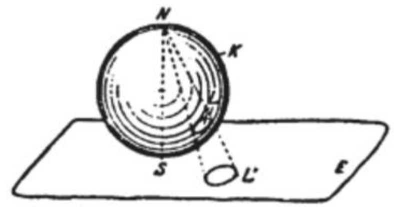


FIG. 2

- **A "foil" for relativistic models**
  Introduce cosmic constant in similar manner

- **<u>Inexact analogy</u>**
  Modified GFE corresponds to P3, not P2

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

- **A significant error?**
  Implications for interpretation
  No interpretation of λ in 1917 paper!

$$\nabla^2\phi + c^2\lambda = 4\pi G\rho \quad \text{(P3)}$$

On the cosmological constant (ii)

- **Schrödinger, 1918**

  Cosmic constant term not necessary for cosmic model

  Introduce negative pressure term in energy-momentum tensor

- **Einstein's reaction**

  New formulation equivalent to original

  (Questionable: physics not the same)

- **Schrödinger, 1918**

  Could pressure term be time-dependent ?

- **Einstein's reaction**

  If not constant, time dependence unknown

  "I have no wish to enter this thicket of hypotheses"

Erwin Schrödinger 1887-1961

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa T_{\mu\nu}$$

$$T_{\mu\nu} = \begin{pmatrix} -p & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & \rho - p \end{pmatrix}$$

On the size of the Einstein World

- **What is the size of the Einstein World?**

  Density of matter from astronomy

  Assume density MW = density of cosmos?

- **Failed to calculate**

  No estimate of cosmic radius in 1917 paper

- **Calculation in correspondence!**

  Takes $\rho = 10^{-22}$ g/cm^3 $\rightarrow$ $R = 10^7$ light-years

  Compares unfavourably with 10^4 light-years (astronomy)

- **Solution to paradox**

  Density of MW $\neq$ density of cosmos

  Challenge for astronomers!

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

Now to our problem! From the standpoint of astronomy, of course, I have erected but a lofty castle in the air.[6] For me, though, it was a burning question whether the relativity concept can be followed through to the finish or whether it leads to contradictions. I am satisfied now that I was able to think the idea through to completion without encountering contradictions. Now I am no longer plagued with the problem, while previously it gave me no peace. Whether the model I formed for myself corresponds to reality is another question, about which we shall probably never gain information. On the value of R, I contemplated the following.[7]

Astronomers have found the spatial density of matter from star counts up to the nth size class, fairly independent of the class to which the count extends, at about

$$10^{-22}\mathrm{g/cm^3}.$$

From this, approximately

$$R = 10^7 \text{ light-years}$$

results, whereas we only see as far as 10^4 light-years. One thing seems strange to me, though. Stars close to our antipodal point should be emitting a lot of light to us.[8] It is doubtful, however, that they could appear point-shaped, since the light velocity varies irregularly. If such a thing were visible in the heavens, it would be noticeable through its negative parallax.[9] We should at least keep an eye out whether any objects with a negative parallax exist in the sky. But now, enough of this, or else you will laugh at me.

EINSTEIN CANNOT MEASURE UNIVERSE

With Mean Density of Matter Unknown the Problem Is Impossible.

FINAL PRINCETON LECTURE

Universe Called Finite and Yet Infinite Because of Its Curved Nature.

On the stability of the Einstein World

- **How does cosmic constant term work?**
  Assume uniform distribution of matter

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

- **Perturbation**
  What happens if the density of matter varies slightly?

- **Failed to consider**
  No mention of issue in 1917
  No mention of issue for many years

- **Lemaître (1927)**
  Cosmos expanding from Einstein World

- **Eddington (1930)**
  Einstein World unstable

668 *Prof. A. S. Eddington,* xc. 7,

On the Instability of Einstein's Spherical World.
By A. S. Eddington, F.R.S.

1. Working in conjunction with Mr. G. C. McVittie, I began some months ago to examine whether Einstein's spherical universe is stable. Before our investigation was complete we learnt of a paper by Abbé G. Lemaître * which gives a remarkably complete solution of the various questions connected with the Einstein and de Sitter cosmogonies. Although not expressly stated, it is at once apparent from his formulæ that the Einstein world is unstable—an important fact which, I think, has not hitherto been appreciated in cosmogonical discussions. Astronomers are deeply interested in these recondite problems owing to their connection with the behaviour of spiral nebulæ ; and I desire to review the situation from an astronomical standpoint, although my original hope of contributing some definitely new result has been forestalled by Lemaître's brilliant solution.

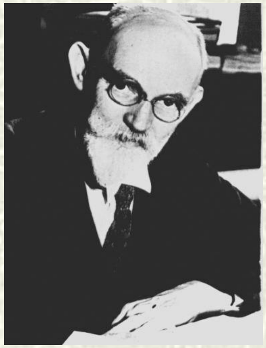
Finitude of space depends on a " cosmical constant " λ, which occurs in Einstein's gravitational equations $G_{\mu\nu} = \lambda g_{\mu\nu}$ for empty space. On general philosophical grounds † there can be little doubt that this form of the equations is correct rather than his earlier form $G_{\mu\nu} = 0$; but λ is so small as to be negligible in all but very large scale applications. Except in so far as a value may be suggested by astronomical survey of the extragalactic universe, λ is unknown ; or it would be better to say that we do not know the lengths of the objects and standards of our

III Einstein and alternate cosmologies

- **An empty universe (de Sitter, 1917)**
  Alternative cosmic solution for the GFE
  Closed curvature of space-time

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G + \lambda g_{\mu\nu} = 0$$

$$\rho = 0; \;\; \lambda = \frac{3}{R^2}$$

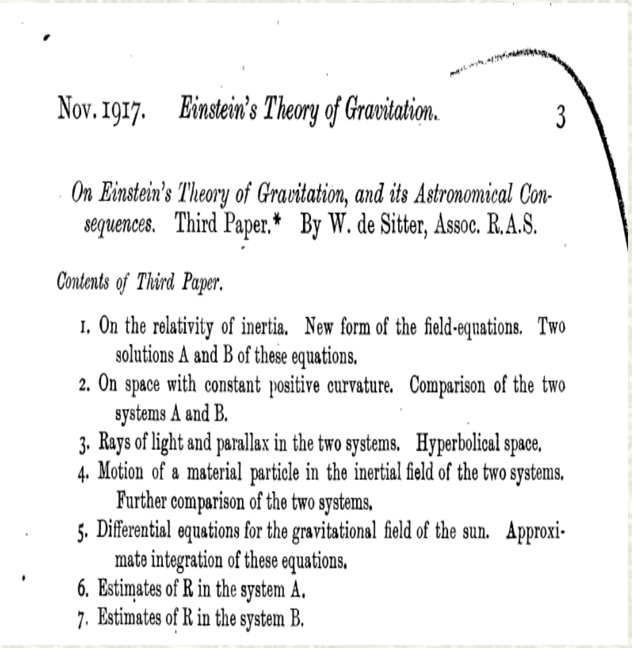


Willem de Sitter

- **Solution B**
  Curvature of space determined by cosmic constant
  Solution enabled by cosmic constant

- **Einstein's reaction**
  Dismay; unrealistic
  Conflict with Mach's principle (doubts about λ?)

- **Interest from astronomers**
  'de Sitter effect'
  Chimed with Slipher's observations of the spiral nebulae



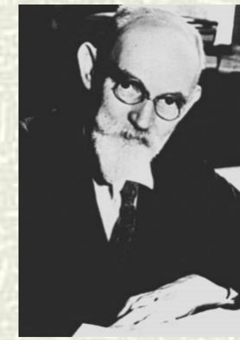
Nov. 1917. *Einstein's Theory of Gravitation.* 3

On Einstein's Theory of Gravitation, and its Astronomical Consequences. Third Paper.* By W. de Sitter, Assoc. R.A.S.

Contents of Third Paper.

1. On the relativity of inertia. New form of the field-equations. Two solutions A and B of these equations.
2. On space with constant positive curvature. Comparison of the two systems A and B.
3. Rays of light and parallax in the two systems. Hyperbolical space.
4. Motion of a material particle in the inertial field of the two systems. Further comparison of the two systems.
5. Differential equations for the gravitational field of the sun. Approximate integration of these equations.
6. Estimates of R in the system A.
7. Estimates of R in the system B.

The Einstein-deSitter-Weyl-Klein debate



- **de Sitter solution disliked by Einstein**

  Conflict with Mach's principle

  Problems with singularities? (1918)

  Lack of singularity conceded (non-static case)

  Considered unrealistic

$$\rho = 0: \ \lambda = \frac{3}{R^2}$$

- **Arguing past each other?**

  Not Machian

  Not static ?

- **A second de Sitter confusion**

  Weyl, Lanczos, Klein, Lemaître

  Static or non-static - a matter of co-ordinates?

[p. 270] 5. "Critical Comment on a Solution of the Gravitational Field Equations Given by Mr. De Sitter"

[*Einstein 1918c*]

SUBMITTED 7 March 1918
PUBLISHED 21 March 1918

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1918): 270–272.

[1] Herr De Sitter, to whom we owe deeply probing investigations into the field of the general theory of relativity, has recently given a solution for the equations of gravitation[1] which, in his opinion, could possibly represent the metric structure of the universe. However, it appears to me that one can raise a grave argument against the admissibility of this solution, which shall be presented in the following.

The De Sitter solution of the field equations

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \kappa T \qquad (1)$$

is

Einstein *vs* Friedman

Alexander Friedman
(1888 -1925)

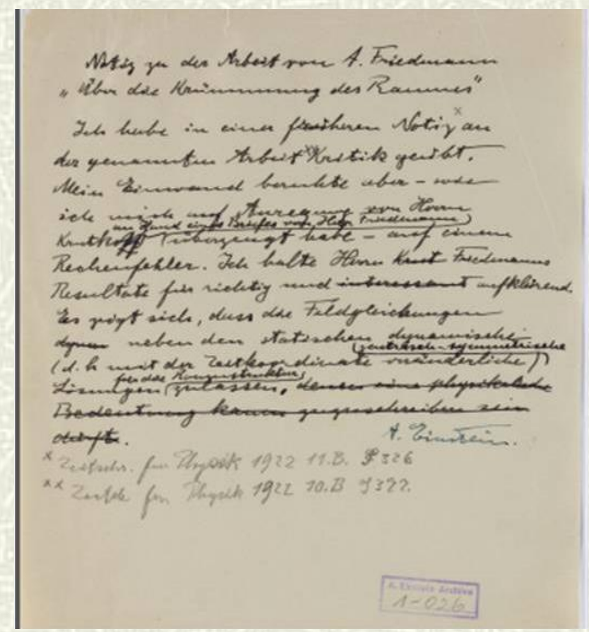
- **Alexander Friedman (1922)**
  Allow time-varying solutions for the cosmos
  Expanding or contracting universe

$$\frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho,$$

$$\frac{R'^2}{R^2} + \frac{2RR''}{R^2} + \frac{c^2}{R^2} - \lambda = 0.$$

- **Evolving universe**
  Time-varying density of matter
  Positive or negative spatial curvature
  Depends on matter $\Omega = d/d_c$

- **Einstein's reaction**
  Declared solution invalid (1922)
  Retracted one year later (1923)
  Hypothetical (unrealistic) solution


Notiz zu der Arbeit von A. Friedmann „Über die Krümmung des Raumes"
Ich habe in einer früheren Notiz an der genannten Arbeit Kritik geübt. Mein Einwand beruhte aber – wie ich mich auf Anregung von Herrn Krutkoff überzeugt habe – auf einem Rechenfehler. Ich halte Herrn Friedmanns Resultate für richtig und aufklärend. Es zeigt sich, dass die Feldgleichungen neben den statischen dynamische (d. h. mit der Zeitkoordinate veränderliche) zentrisch-symmetrische Lösungen für die Raumstruktur zulassen, denen eine physikalische Bedeutung kaum zuzuschreiben sein dürfte.
A. Einstein.
x Zeitschr. für Physik 1922 11.B. S. 326
xx Zeitschr. für Physik 1922 10.B S. 377.

"~~To this a physical reality can hardly be ascribed~~"

Einstein *vs* Lemaître

- **Georges Lemaître (1927)**
  Allow time-varying solutions (expansion)
  Retain cosmic constant

- **Inspired by astronomical observation**
  Redshifts of the nebulae (Slipher)
  Extra-galactic nature of the nebulae (Hubble)

- **Expansion from static Einstein World**
  Instability (implicit)

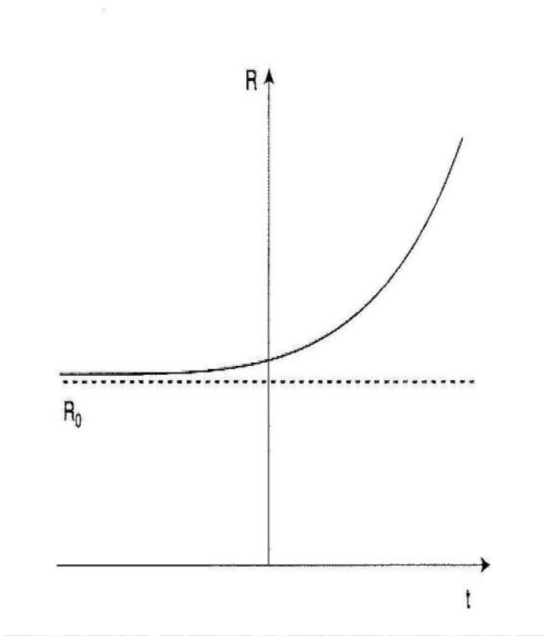
- **Einstein's reaction**
  Expanding models "abominable" (conversation)

Einstein not au fait with astronomy?

$$\frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho,$$

$$\frac{R'^2}{R^2} + \frac{2RR''}{R^2} + \frac{c^2}{R^2} - \lambda = 0.$$

Georges Lemaître (1894-1966)



A watershed in cosmology

Edwin Hubble (1889-1953)

- **Hubble's law (1929)**

  A redshift/distance relation for the spiral nebulae

  Linear relation: $h = 500\ kms^{-1}Mpc^{-1}$

- **Evidence of cosmic expansion?**

  RAS meeting (1930): Eddington, de Sitter

- **Friedman-Lemaître models circulated**

  Time-varying radius and density of matter

- **Einstein apprised**

  Cambridge visit (June 1930)

  Sojourn at Caltech (Spring 1931)

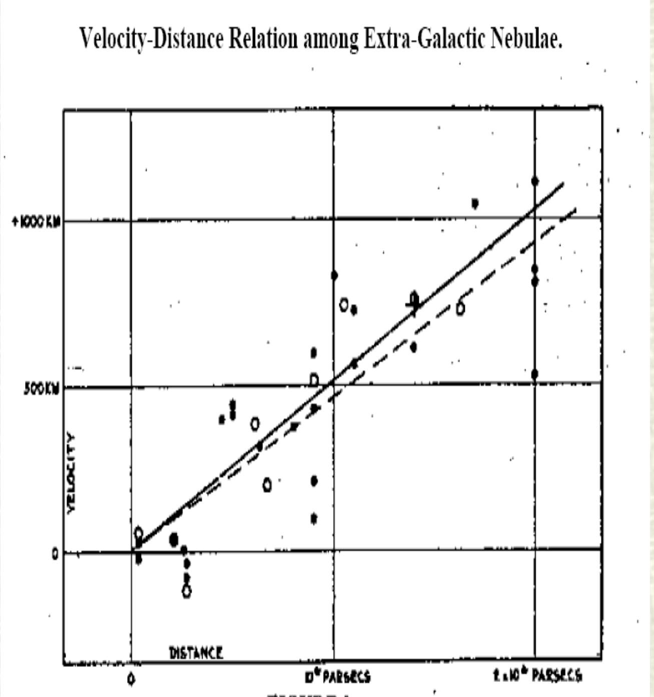


FIGURE 1

The expanding universe (1930-32)

- **Eddington (1930, 31)**

  On the instability of the Einstein universe
  Expansion caused by condensation?

- **Tolman (1930, 31)**

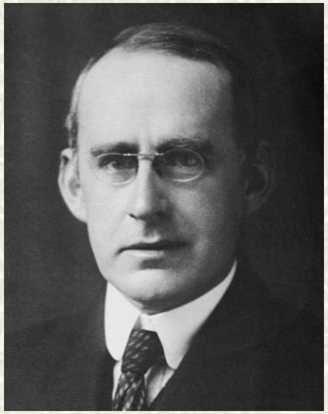
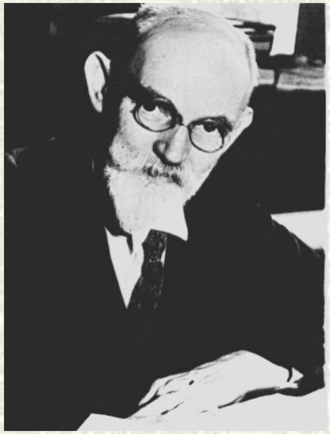
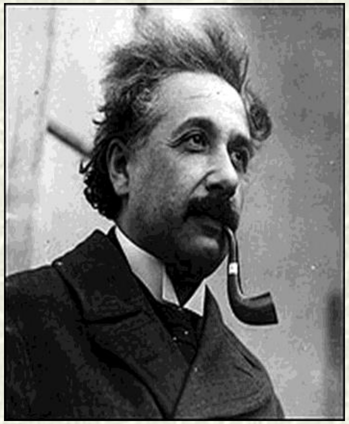
  On the behaviour of non-static models
  Expansion caused by annihilation of matter ?

- **de Sitter (1930, 31)**

  Further remarks on the expanding universe
  Expanding universes of every flavour

- **Einstein (1931, 32)**

  Friedman-Einstein model $k = 1,\ \lambda = 0$
  Einstein-de Sitter model $k = 0,\ \lambda = 0$

Expanding models

Einstein's steady–state model (~1931): λ = energy of the vacuum?

Einstein’s steady-state model (~1931)

- **Unpublished manuscript**

  Archived as draft of Friedman-Einstein model

  Similar title, opening

- **Steady-state model**

  “The density is constant and determines the expansion”

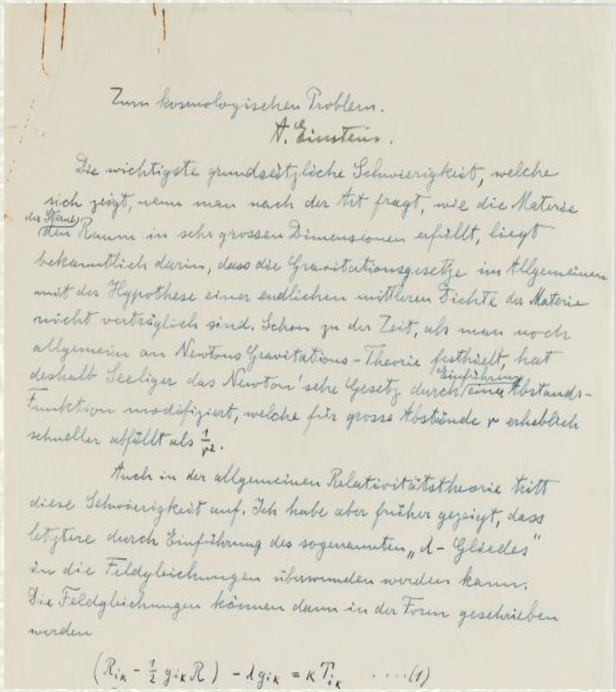
  Associates creation of matter with λ

- **Fatal flaw**

  Null solution

  Abandoned in favour of evolving models

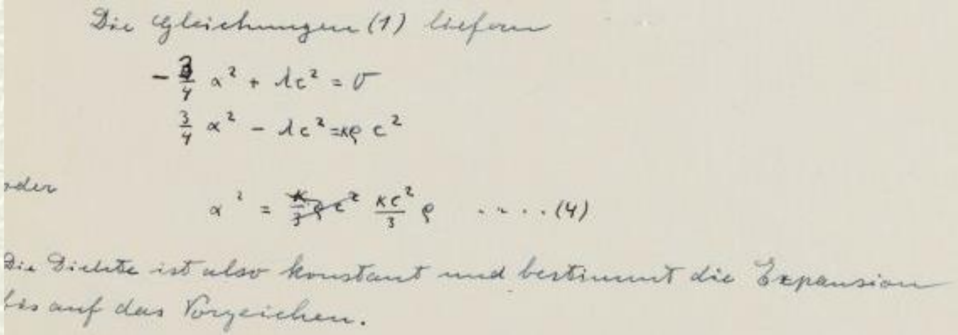
O’Raifeartaigh *et al.* 2016
Nussbaumer 2016


Zum kosmologischen Problem.
A. Einstein.

Die wichtigste grundsätzliche Schwierigkeit, welche sich zeigt, wenn man nach der Art fragt, wie die Materie der Sterne den Raum in sehr grossen Dimensionen erfüllt, liegt bekanntlich darin, dass die Gravitationsgesetze im Allgemeinen mit der Hypothese einer endlichen mittleren Dichte der Materie nicht verträglich sind. Schon zu der Zeit, als man noch allgemein an Newtons Gravitations-Theorie festhielt, hat deshalb Seeliger das Newton'sche Gesetz durch Einführung einer Abstands-Funktion modifiziert, welche für grosse Abstände r erheblich schneller abfällt als $\frac{1}{r^2}$.

Auch in der allgemeinen Relativitätstheorie tritt diese Schwierigkeit auf. Ich habe aber früher gezeigt, dass letztere durch Einführung des sogenannten „λ-Gliedes“ in die Feldgleichungen überwunden werden kann. Die Feldgleichungen können dann in der Form geschrieben werden

$$\left(R_{ik} - \frac{1}{2} g_{ik} R\right) - \lambda g_{ik} = \kappa T_{ik} \quad \ldots (1)$$


Die Gleichungen (1) liefern

$$-\frac{3}{4}\alpha^2 + \lambda c^2 = 0$$
$$\frac{3}{4}\alpha^2 - \lambda c^2 = \kappa\rho c^2$$

oder

$$\alpha^2 = \frac{\kappa c^2}{3}\rho \quad \ldots (4)$$

Die Dichte ist also konstant und bestimmt die Expansion bis auf das Vorzeichen.

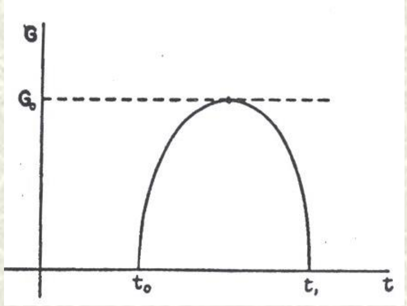
The Friedman-Einstein model (1931)

- **Cosmic constant abandoned**

  Unsatisfactory (unstable solution)

  Unnecessary (non-static universe)

$$\left(\frac{dP}{dt}\right)^2 = c^2\frac{P_0 - P}{P}$$

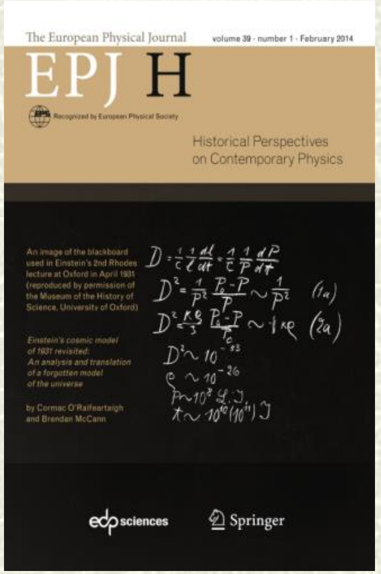


$$P \sim \frac{1}{D} \qquad D^2 \sim \kappa\rho$$

- **Calculations of cosmic radius and density**

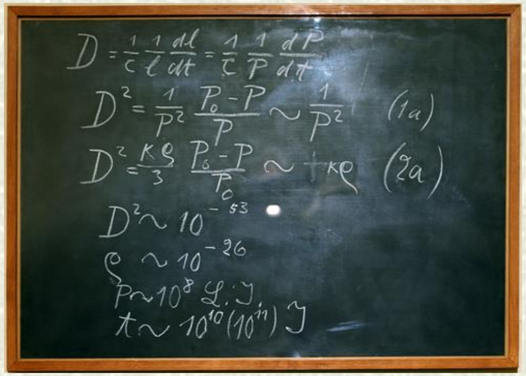
  Einstein: $P \sim 10^8$ lyr, $\rho \sim 10^{-26}$ g/cm^3 , $t \sim 10^{10}$ yr

  We get: $P \sim 10^9$ lyr, $\rho \sim 10^{-28}$ g/cm^3 , $t \sim 10^9$ yr



- **Explanation for age paradox?**

  Assumption of homogeneity at early epochs



- **<u>Not a cyclic model</u>**

  "Model fails at P = 0 "

  Contrary to what is usually stated

Einstein-de Sitter model (1932)

- **Curvature not a given in dynamic models**

  Not observed empirically

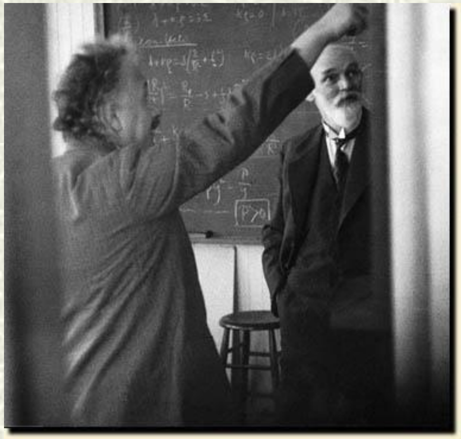
  Remove spatial curvature (Occam's razor)

$$ds^2 = -R^2(dx^2 + dy^2 + dz^2) + c^2dt^2$$

$$\frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho,$$

$$\frac{1}{R^2}\left(\frac{dR}{cdt}\right)^2 = \frac{1}{3}\kappa\rho.$$

$$h^2 = \frac{1}{3}\kappa\rho$$



- **Simplest Friedman model**

  Time-varying universe with $\lambda = 0$, $k = 0$

  Important hypothetical case: critical universe

  Critical density : $\rho = 10^{-28}$ g/cm^3

- **<u>Becomes standard model</u>**

  Despite high density of matter

  Despite age problem

<u>Time evolution not considered in paper – see title</u>

PROCEEDINGS
OF THE
NATIONAL ACADEMY OF SCIENCES
Volume 18 March 15, 1932 Number 3

ON THE RELATION BETWEEN THE EXPANSION AND THE MEAN DENSITY OF THE UNIVERSE

By A. Einstein and W. de Sitter

Communicated by the Mount Wilson Observatory, January 25, 1932

In a recent note in the *Göttinger Nachrichten*, Dr. O. Heckmann has pointed out that the non-static solutions of the field equations of the general theory of relativity with constant density do not necessarily imply a positive curvature of three-dimensional space, but that this curvature may also be negative or zero.

“My greatest blunder”

- **Einstein’s description of cosmic constant term**

  Reported by George Gamow

- **Controversy**

  Queried by Straumann, Livio

  Not in Einstein’s papers or other reports

- **Our findings**

  Consistent with actions

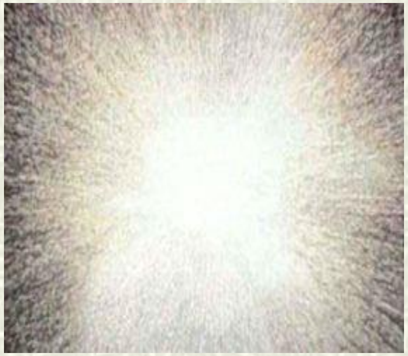
  Einstein’s remark reported by Gamow, Alpher, Wheeler

- **Meaning of remark**

  Failure to spot instability of static solution

  Failure to predict expanding universe

Georges Gamow



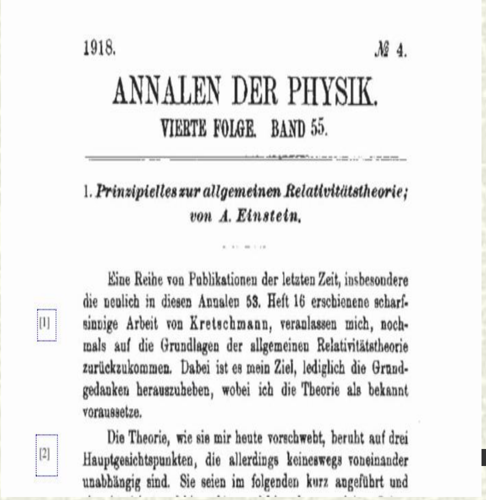
Conclusions

- **Historical aspects of 1917 paper**
  Continuation of relativity project

- **Philosophical aspects of 1917 paper**
  Inspired by Mach's principle

- **Assumptions**
  Non-zero mean density of matter (uniform)
  Static universe (observation)
  Failure to spot instability of static solution

- **New evidence**
  Happy to embrace expanding universe
  Minimal models - Occam's razor



1918. № 4.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 55.

1. *Prinzipielles zur allgemeinen Relativitätstheorie;*
von A. Einstein.

[1] Eine Reihe von Publikationen der letzten Zeit, insbesondere die neulich in diesen Annalen 53. Heft 16 erschienene scharfsinnige Arbeit von Kretschmann, veranlassen mich, nochmals auf die Grundlagen der allgemeinen Relativitätstheorie zurückzukommen. Dabei ist es mein Ziel, lediglich die Grundgedanken herauszuheben, wobei ich die Theorie als bekannt voraussetze.

[2] Die Theorie, wie sie mir heute vorschwebt, beruht auf drei Hauptgesichtspunkten, die allerdings keineswegs voneinander unabhängig sind. Sie seien im folgenden kurz angeführt und

No mention of origins

Coda: The Einstein World today

- **The question of origins**

  BB model ≠ a theory of origins

  The singularity problem

  The quantum gravity problem

- <u>**The cyclic universe**</u>

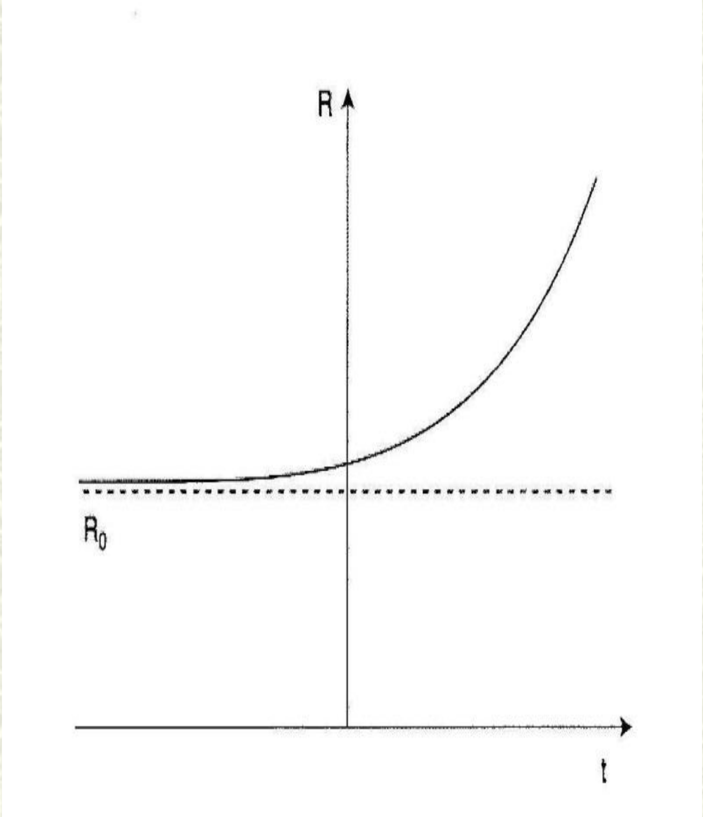
  From BC to BB

- <u>**The emergent universe**</u>

  Inflating from a static Einstein World

- **On the stability of the Einstein World**

  Advanced GR: LQG, DGR, B-D, f(R), f (R,T)



Relevance of past theories in modern science

Physics > History and Philosophy of Physics

Einstein's 1917 Static Model of the Universe: A Centennial Review

Cormac O'Raifeartaigh, Michael O'Keeffe, Werner Nahm, Simon Mitton

(Submitted on 25 Jan 2017 (v1), last revised 10 May 2017 (this version, v2))

We present a historical review of Einstein's 1917 paper 'Cosmological Considerations in the General Theory of Relativity' to mark the centenary of a key work that set the foundations of modern cosmology. We find that the paper followed as a natural next step after Einstein's development of the general theory of relativity and that the work offers many insights into his thoughts on relativity, astronomy and cosmology. Our review includes a description of the observational and theoretical background to the paper; a paragraph-by-paragraph guided tour of the work; a discussion of Einstein's views of issues such as the relativity of inertia, the curvature of space and the cosmological constant. Particular attention is paid to little-known aspects of the paper such as Einstein's failure to test his model against observation, his failure to consider the stability of the model and a mathematical oversight concerning his interpretation of the role of the cosmological constant. We recall the response of theorists and astronomers to Einstein's cosmology in the context of the alternate models of the universe proposed by Willem de Sitter, Alexander Friedman and Georges Lemaitre. Finally, we describe the relevance of the Einstein World in today's 'emergent' cosmologies.

Comments: Revised version of paper with some edits and corrections. Accepted for publication in the European Physical Journal (H)

Subjects: **History and Philosophy of Physics (physics.hist-ph)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO)

Cite as: arXiv:1701.07261 **[physics.hist-ph]**

(or arXiv:1701.07261v2 **[physics.hist-ph]** for this version)

The Friedman-Einstein model

- **First translation into English**

  O'Raifeartaigh and McCann 2014

- **<u>Not a cyclic model</u>**

  "Model fails at P = 0 "

  Contrary to what is usually stated

- **Anomalies in calculations of radius and density**

  Einstein: P ~ 10^{8} lyr, $\rho \sim 10^{-26}$ g/cm^{3} , $t \sim 10^{10}$ yr

  We get: $P \sim 10^{9}$ lyr, $\rho \sim 10^{-28}$ g/cm^{3} , $t \sim 10^{9}$ yr

- **Source of error?**

  Oxford blackboard: $D^2 \sim 10^{-53}$ cm^{-2} *should be* 10^{-55} cm^{-2}

  Time miscalculation $t \sim 10^{10}$ yr (*should be* 10^{9} yr)

  Non-trivial error: misses conflict with radioactivity