The Einstein World

A Centennial Review of
Einstein's 1917 Static Model of the Universe

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Irish Quantum Foundations Conference 2017

Overview

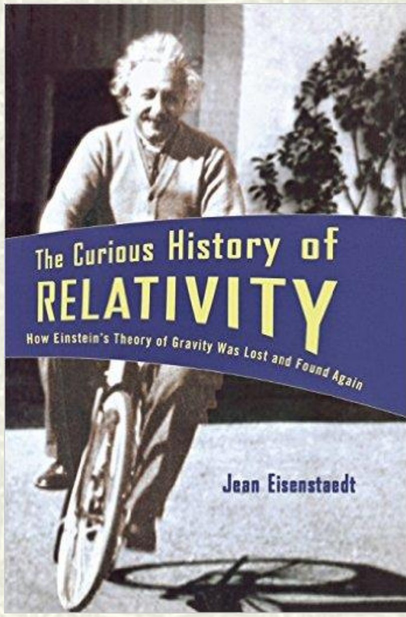
- **Historical context**
  Biographical context of the paper
  Scientific context of the paper

- **Einstein's 1917 model of the cosmos**
  Basic assumptions
  The cosmological constant term
  Theoretical and empirical issues

- **Reception**
  The de Sitter model
  The non-static cosmologies of Friedman and Lemaître

Coda: The Einstein World today

Einstein in Berlin (1916)



Historical context

- **Appointed to Berlin Chair**

  Arrives April 1914

  Family leave Berlin, June 1914

- **World War I (1914-18)**

  Living alone, food shortages

  Dietary problems, illness

- **Second 'miraculous' period**

  Covariant field equations (1915)

  Exposition, solutions and predictions (1916)

  First relativistic model of the cosmos (1917)

  Papers on gravitational waves

  Papers on the quantum theory of radiation

  Papers on unified field theory

Einstein in Berlin (1916)

The road to general relativity

- **The general principle of relativity (1907-)**

  Relativity and accelerated motion?

- **The principle of equivalence**

  Equivalence of gravity and acceleration

- **The principle of Mach**

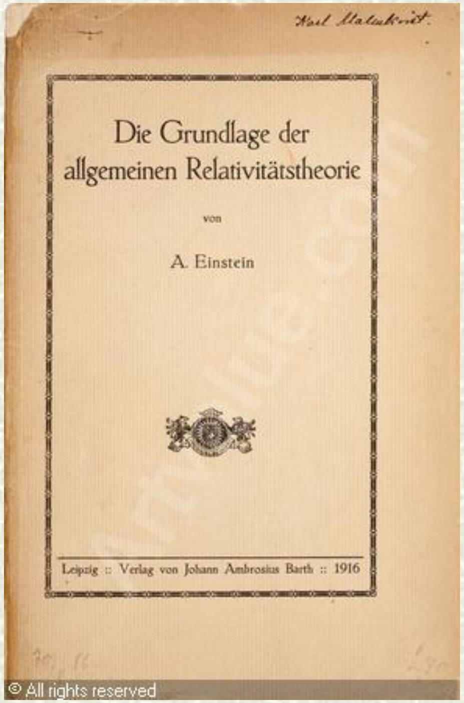
  Relativity of inertia

  Structure of space determined by matter

- **A long road (1907-1915)**

  Gravity = curvature of space-time

  Covariant field equations?



$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa T_{\mu\nu}$$

$$G_{\mu\nu} = -\kappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right)$$

Relativistic cosmology (1915-17)

- **A natural progression**

  Ultimate test for new theory of gravitation

- **Principle 1: stasis**

  Assume static distribution of matter

- **Principle 2: uniformity**

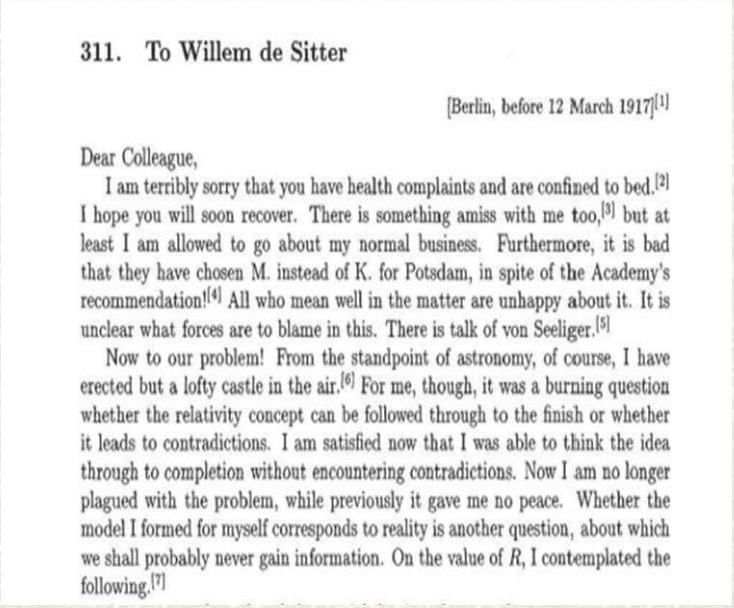
  Assume uniform distribution of matter

- **Principle 3: Mach's principle**

  No such thing as empty space

- **Boundary conditions at infinity?**

  What are the 'natural' values of the $g_{\mu\nu}$?



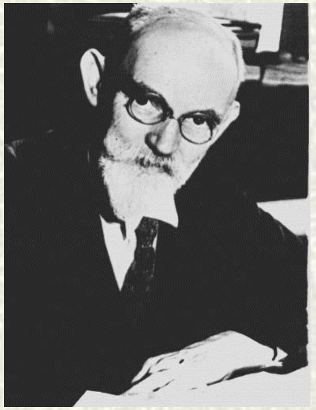
311. To Willem de Sitter

[Berlin, before 12 March 1917][1]

Dear Colleague,

I am terribly sorry that you have health complaints and are confined to bed.[2] I hope you will soon recover. There is something amiss with me too,[3] but at least I am allowed to go about my normal business. Furthermore, it is bad that they have chosen M. instead of K. for Potsdam, in spite of the Academy's recommendation![4] All who mean well in the matter are unhappy about it. It is unclear what forces are to blame in this. There is talk of von Seeliger.[5]

Now to our problem! From the standpoint of astronomy, of course, I have erected but a lofty castle in the air.[6] For me, though, it was a burning question whether the relativity concept can be followed through to the finish or whether it leads to contradictions. I am satisfied now that I was able to think the idea through to completion without encountering contradictions. Now I am no longer plagued with the problem, while previously it gave me no peace. Whether the model I formed for myself corresponds to reality is another question, about which we shall probably never gain information. On the value of R, I contemplated the following.[7]



Cosmological Considerations (1917)

Doc. 43

Cosmological Considerations in the General Theory of Relativity

[1]

This translation by W. Perrett and G. B. Jeffery is reprinted from H. A. Lorentz et al., *The Principle of Relativity* (Dover, 1952), pp. 175–188.

It is well known that Poisson's equation

$$\nabla^2\phi = 4\pi K\rho \quad . \quad . \quad . \quad . \quad (1)$$

in combination with the equations of motion of a material point is not as yet a perfect substitute for Newton's theory of action at a distance. There is still to be taken into account the condition that at spatial infinity the potential ϕ tends toward a fixed limiting value. There is an analogous state of things in the theory of gravitation in general relativity. Here, too, we must supplement the differential equations by limiting conditions at spatial infinity, if we really have to regard the universe as being of infinite spatial extent. [2]

In my treatment of the planetary problem I chose these limiting conditions in the form of the following assumption: it is possible to select a system of reference so that at spatial infinity all the gravitational potentials $g_{\mu\nu}$ become constant. [3] But it is by no means evident *a priori* that we may lay down the same limiting conditions when we wish to take larger portions of the physical universe into consideration. In the following pages the reflexions will be given which, up to the present, I have made on this fundamentally important question.

§ 1. The Newtonian Theory

It is well known that Newton's limiting condition of the constant limit for ϕ at spatial infinity leads to the view that the density of matter becomes zero at infinity. For we imagine that there may be a place in universal space round about which the gravitational field of matter, viewed on a large scale, possesses spherical symmetry. It then follows from Poisson's equation that, in order that ϕ may tend to a

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\varrho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential ϕ einem festen Grenzwerte zustrebt. Analog verhält es sich bei der Gravitationstheorie der allgemeinen Relativität; auch hier müssen zu den Differentialgleichungen Grenzbedingungen hinzutreten für das räumlich Unendliche, falls man die Welt wirklich als räumlich unendlich ausgedehnt anzusehen hat.

Bei der Behandlung des Planetenproblems habe ich diese Grenzbedingungen in Gestalt folgender Annahme gewählt: Es ist möglich, ein Bezugssystem so zu wählen, daß sämtliche Gravitationspotentiale $g_{\mu\nu}$ im räumlich Unendlichen konstant werden. Es ist aber a priori durchaus nicht evident, daß man dieselben Grenzbedingungen ansetzen darf, wenn man größere Partien der Körperwelt ins Auge fassen will. Im folgenden sollen die Überlegungen angegeben werden, welche ich bisher über diese prinzipiell wichtige Frage angestellt habe.

§ 1. Die Newtonsche Theorie.

Es ist wohlbekannt, daß die Newtonsche Grenzbedingung des konstanten Limes für ϕ im räumlich Unendlichen zu der Auffassung hinführt, daß die Dichte der Materie im Unendlichen zu null wird. Wir denken uns nämlich, es lasse sich ein Ort im Weltraum finden, um den herum das Gravitationsfeld der Materie, im großen betrachtet, Kugelsymmetrie besitzt (Mittelpunkt). Dann folgt aus der Poissonschen Gleichung, daß die mittlere Dichte ϱ rascher als $\frac{1}{r^2}$ mit wachsender Entfernung r vom Mittelpunkt zu null herabsinken muß, damit ϕ im

Structure of Einstein's 1917 paper

§ 1. The Newtonian Theory

§ 2. The Boundary Conditions According to the General Theory of Relativity

§ 3. The Spatially Finite Universe with a Uniform Distribution of Matter

$$G_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

§ 4. On an Additional Term for the Field Equations of Gravitation

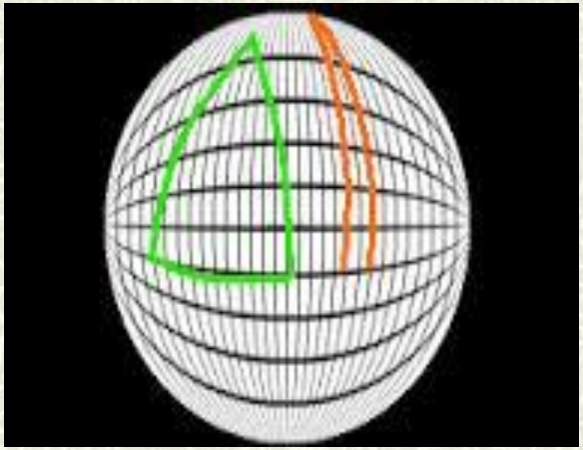
$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

§ 5. Calculation and Result

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

The Einstein World

- **Assume stasis** *(no evidence to the contrary)*
  Non-zero density of matter



- **Introduce closed spatial curvature**
  To conform with Mach's principle
  Solves problem of $g_{\mu\nu}$

- **Introduce new term in GFE***
  Additional term needed in field equations

$$G_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

- **Quantitative model of the universe**
  Radius related to matter density
  Radius related to cosmic constant

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

On the cosmological constant (i)

From 3(a), in accordance with (1a) one calculates for the $R_{\mu\nu}$ $(x_1 = x_2 = x_3 = 0)$ the values

$$\begin{matrix} -\frac{2}{P^2} & 0 & 0 & 0 \\ 0 & -\frac{2}{P^2} & 0 & 0 \\ 0 & 0 & -\frac{2}{P^2} & 0 \\ 0 & 0 & 0 & 0 \end{matrix},$$

for $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, the values

$$\begin{matrix} \frac{1}{P^2} & 0 & 0 & 0 \\ 0 & \frac{1}{P^2} & 0 & 0 \\ 0 & 0 & \frac{1}{P^2} & 0 \\ 0 & 0 & 0 & -\frac{3c^2}{P^2} \end{matrix},$$

while for $-\kappa \boldsymbol{T}$ one obtains the values

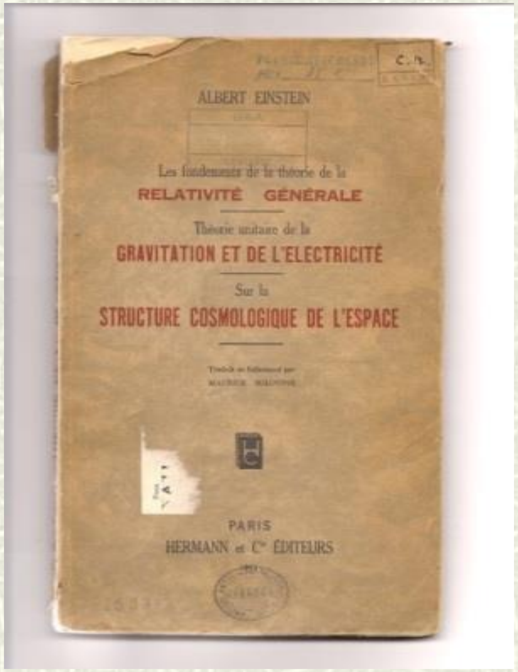
$$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\kappa\rho c^2 \end{matrix}$$

Thus from (1) the two contradictory equations are obtained

$$\left.\begin{aligned} \frac{1}{P^2} &= 0 \\ \frac{3c^2}{P^2} &= \kappa\rho c^2 \end{aligned}\right\} \quad (4)$$

$$\boldsymbol{G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa\, T_{\mu\nu}}$$

$$ds^2 = \frac{dx_1^2 + dx_2^2 + dx_3^2}{\left(1 + \frac{r^2}{(2P)^2}\right)^2} - c^2 dt^2$$



Einstein 1933

λ term needed for (static) solution

On the cosmological constant (ii)

- **Introduced in analogy with Newtonian cosmology**

  Full section on Newtonian gravity (Einstein 1917)

  Indefinite potential at infinity?

$$\nabla^2\phi = 4\pi G\rho \quad \text{(P1)}$$

$$\nabla^2\phi - \lambda\phi = 4\pi G\rho \quad \text{(P2)}$$

- **Modifying Newtonian gravity**

  Extra term in Poisson's equation

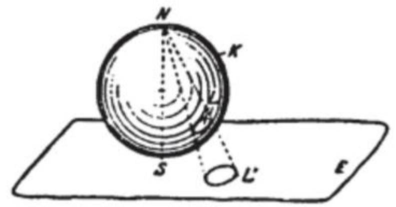


FIG. 2

- **A "foil" for relativistic models**

  Introduce cosmic constant in similar manner

- **<u>Inexact analogy</u>**

  Modified GFE corresponds to P3, not P2

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

- **A significant error?**

  Implications for interpretation

$$\nabla^2\phi + c^2\lambda = 4\pi G\rho \quad \text{(P3)}$$

On the cosmological constant (iii)

- **Schrödinger, 1918**

  Cosmic constant term not necessary for cosmic model

  Negative pressure term in energy-momentum tensor

- **Einstein's reaction**

  New formulation equivalent to original

  (Questionable: physics not the same)

- **Schrödinger, 1918**

  Could pressure term be time-dependent ?

- **Einstein's reaction**

  If not constant, time dependence unknown

  "I have no wish to enter this thicket of hypotheses"

Erwin Schrödinger 1887-1961

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G = -\kappa T_{\mu\nu}$$

$$T_{\mu\nu} = \begin{pmatrix} -p & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & \rho - p \end{pmatrix}$$

The size of the Einstein World

- **What is the size of the Einstein World?**
  Assume uniform distribution of matter

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

- **Density of matter**
  Density of matter in MW from astronomy
  Assume density MW = density of cosmos?

- **Failed to calculate**
  No estimate of cosmic radius in 1917 paper
  Declares density unknown in reviews

- **Calculation in correspondence!**
  Takes $\rho = 10^{-22}$ g/cm^3 $\rightarrow$ $R = 10^7$ light-years
  Compares unfavourably with 10^4 light-years (astronomy)

Now to our problem! From the standpoint of astronomy, of course, I have erected but a lofty castle in the air.[6] For me, though, it was a burning question whether the relativity concept can be followed through to the finish or whether it leads to contradictions. I am satisfied now that I was able to think the idea through to completion without encountering contradictions. Now I am no longer plagued with the problem, while previously it gave me no peace. Whether the model I formed for myself corresponds to reality is another question, about which we shall probably never gain information. On the value of R, I contemplated the following.[7]

Astronomers have found the spatial density of matter from star counts up to the nth size class, fairly independent of the class to which the count extends, at about

$$10^{-22}\text{g/cm}^3.$$

From this, approximately

$$R = 10^7 \text{ light-years}$$

results, whereas we only see as far as 10^4 light-years. One thing seems strange to me, though. Stars close to our antipodal point should be emitting a lot of light to us.[8] It is doubtful, however, that they could appear point-shaped, since the light velocity varies irregularly. If such a thing were visible in the heavens, it would be noticeable through its negative parallax.[9] We should at least keep an eye out whether any objects with a negative parallax exist in the sky. But now, enough of this, or else you will laugh at me.

EINSTEIN CANNOT MEASURE UNIVERSE

With Mean Density of Matter Unknown the Problem Is Impossible.

FINAL PRINCETON LECTURE

Universe Called Finite and Yet Infinite Because of Its Curved Nature.

The stability of the Einstein World

- **How does cosmic constant term work?**

  Assume uniform distribution of matter

$$\lambda = \frac{\kappa\rho}{2} = \frac{1}{R^2}$$

- **Perturbation**

  What happens if the density of matter varies slightly?

- **Failed to investigate**

  No mention of issue in 1917

  No mention of issue until 1927, 1930

- **Lemaître (1927)**

  Cosmos expanding from Einstein World

- **Eddington (1930)**

  Einstein World unstable

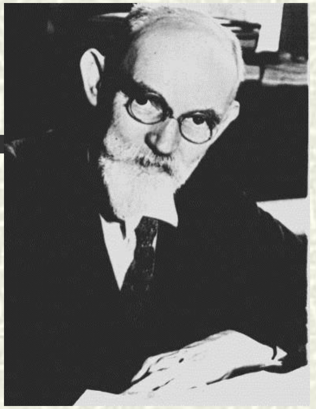
668 *Prof. A. S. Eddington,* XC. 7,

On the Instability of Einstein's Spherical World.
By A. S. Eddington, F.R.S.

1. Working in conjunction with Mr. G. C. McVittie, I began some months ago to examine whether Einstein's spherical universe is stable. Before our investigation was complete we learnt of a paper by Abbé G. Lemaître * which gives a remarkably complete solution of the various questions connected with the Einstein and de Sitter cosmogonies. Although not expressly stated, it is at once apparent from his formulæ that the Einstein world is unstable—an important fact which, I think, has not hitherto been appreciated in cosmogonical discussions. Astronomers are deeply interested in these recondite problems owing to their connection with the behaviour of spiral nebulæ; and I desire to review the situation from an astronomical standpoint, although my original hope of contributing some definitely new result has been forestalled by Lemaître's brilliant solution.

Finitude of space depends on a "cosmical constant" λ, which occurs in Einstein's gravitational equations $G_{\mu\nu} = \lambda g_{\mu\nu}$ for empty space. On general philosophical grounds † there can be little doubt that this form of the equations is correct rather than his earlier form $G_{\mu\nu} = 0$; but λ is so small as to be negligible in all but very large scale applications. Except in so far as a value may be suggested by astronomical survey of the extragalactic universe, λ is unknown; or it would be better to say that we do not know the lengths of the objects and standards of our

Reactions to the Einstein World



Willem de Sitter

- **Alternative cosmic solution for the GFE**

  A universe empty of matter (1917)

  Closed curvature of space-time

$$G_{\mu\nu} - \frac{1}{2} g_{\mu\nu} G + \lambda g_{\mu\nu} = 0$$

$$\rho = 0; \; \lambda = \frac{3}{R^2}$$

- **Solution B**

  Solution enabled by cosmic constant

  Curvature of space determined by cosmic constant

- **Einstein's reaction**

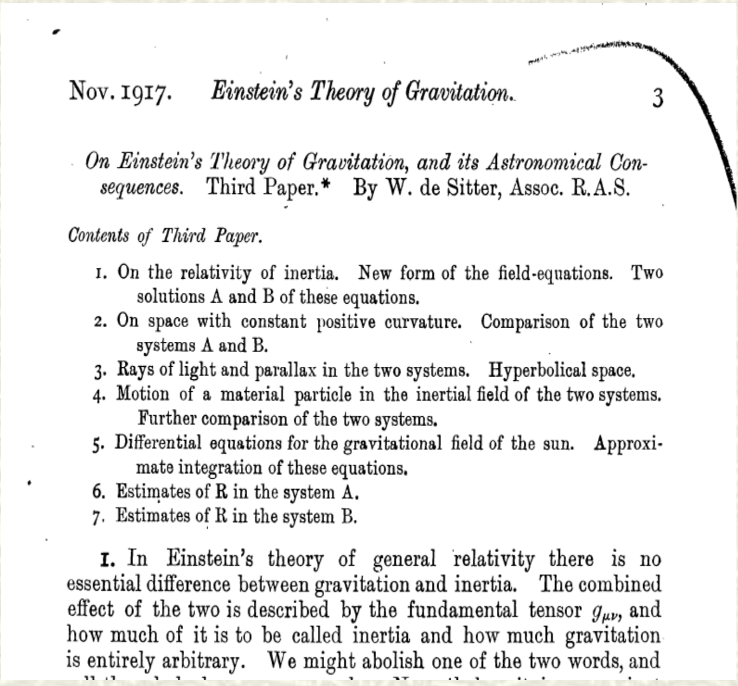
  Unrealistic

  Conflict with Mach's principle

- **Interest from astronomers**

  Light from star would be redshifted

  Chimed with Slipher's observations of the spiral nebulae



Nov. 1917. *Einstein's Theory of Gravitation.* 3

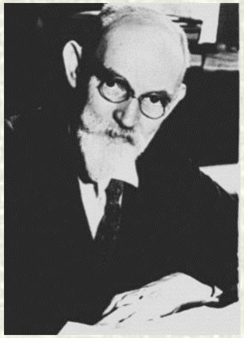
On Einstein's Theory of Gravitation, and its Astronomical Consequences. Third Paper.* By W. de Sitter, Assoc. R.A.S.

Contents of Third Paper.

1. On the relativity of inertia. New form of the field-equations. Two solutions A and B of these equations.
2. On space with constant positive curvature. Comparison of the two systems A and B.
3. Rays of light and parallax in the two systems. Hyperbolical space.
4. Motion of a material particle in the inertial field of the two systems. Further comparison of the two systems.
5. Differential equations for the gravitational field of the sun. Approximate integration of these equations.
6. Estimates of R in the system A.
7. Estimates of R in the system B.

1. In Einstein's theory of general relativity there is no essential difference between gravitation and inertia. The combined effect of the two is described by the fundamental tensor $g_{\mu\nu}$, and how much of it is to be called inertia and how much gravitation is entirely arbitrary. We might abolish one of the two words, and

The Einstein-deSitter-Weyl-Klein debate



- **de Sitter solution disliked by Einstein**

  Conflict with Mach's principle

  Problems with singularities? (1918)

  Lack of singularity conceded

  Considered unrealistic

$$\rho = 0: \ \lambda = \frac{3}{R^2}$$

- **Arguing past each other?**

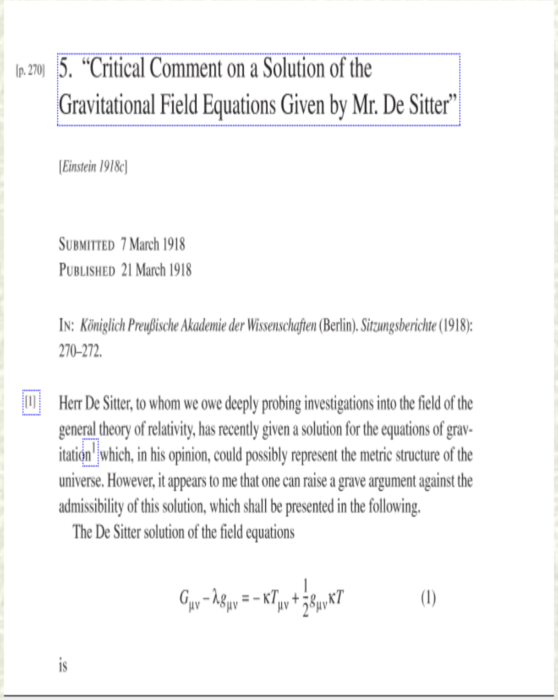
  Not Machian

  Not static ?

- **The de Sitter confusion**

  Weyl, Lanczos, Klein, Lemaître

  Static or non-static - a matter of co-ordinates?



[p. 270] 5. "Critical Comment on a Solution of the Gravitational Field Equations Given by Mr. De Sitter"

[*Einstein 1918c*]

SUBMITTED 7 March 1918
PUBLISHED 21 March 1918

IN: *Königlich Preußische Akademie der Wissenschaften* (Berlin). *Sitzungsberichte* (1918): 270–272.

[1] Herr De Sitter, to whom we owe deeply probing investigations into the field of the general theory of relativity, has recently given a solution for the equations of gravitation[1] which, in his opinion, could possibly represent the metric structure of the universe. However, it appears to me that one can raise a grave argument against the admissibility of this solution, which shall be presented in the following.

The De Sitter solution of the field equations

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\kappa T_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \kappa T \qquad (1)$$

is

Non-static cosmologies

- **Alexander Friedman (1922)**

  *Allow time-varying solutions for the cosmos**

  Two differential equations for R

$$\frac{3R'^2}{R^2} + \frac{3c^2}{R^2} - \lambda = \kappa c^2 \rho,$$

$$\frac{R'^2}{R^2} + \frac{2RR''}{R^2} + \frac{c^2}{R^2} - \lambda = 0.$$

Alexander Friedman (1888 -1925)

- **Evolving universe**

  Time-varying radius and density of matter

  Considered 'suspicious' by Einstein

Georges Lemaître (1894-1966)

- **Georges Lemaître (1927)**

  Theoretical universe of time-varying radius

  Expanding universe in agreement with emerging astronomical data

  Also rejected by Einstein

"Vôtre physique est abominable"

A watershed in cosmology

Edwin Hubble (1889-1953)

- **Hubble's law (1929)**

  A redshift/distance relation for the spiral nebulae

  Linear relation: h = 500 $kms^{-1}Mpc^{-1}$

- **Evidence of cosmic expansion?**

  RAS meeting (1930): Eddington, de Sitter

- **Friedman-Lemaître models circulated**

  Time-varying radius and density of matter

- **Einstein apprised**

  Cambridge visit (June 1930)

  Sojourn at Caltech (Spring 1931)

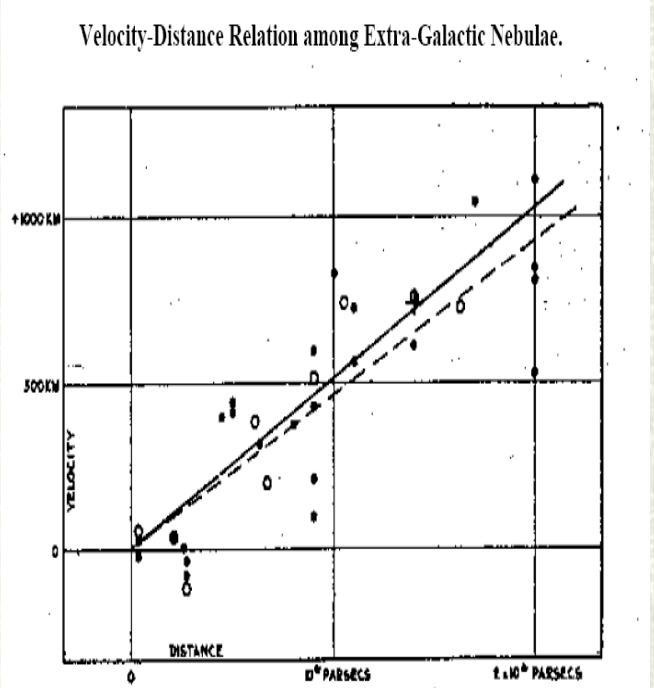


FIGURE 1

The expanding universe (1930 -)

- **Eddington (1930, 31)**

  On the instability of the Einstein universe

  Expansion caused by condensation?

- **Tolman (1930, 31)**

  On the behaviour of non-static models

  Expansion caused by annihilation of matter ?

- **de Sitter (1930, 31)**

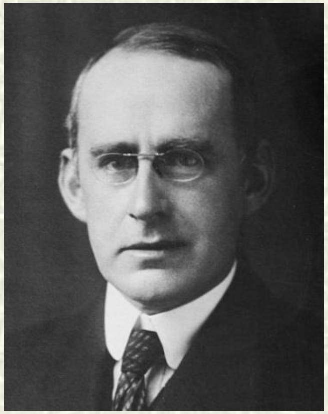
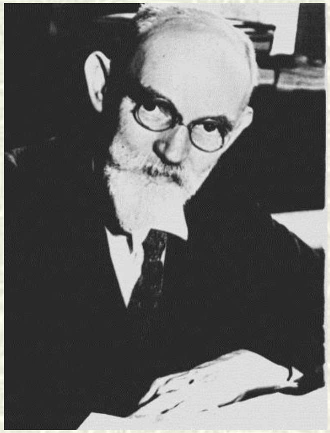
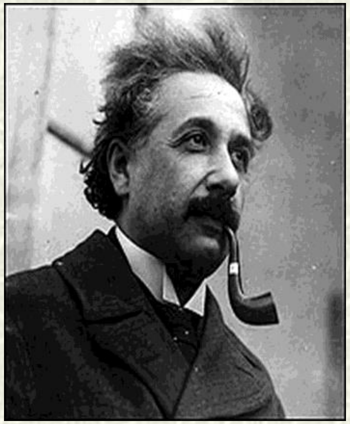
  Further remarks on the expanding universe

  Expanding universes of every flavour

- **Einstein (1931, 32)**

  Friedman-Einstein model $k = 1, \lambda = 0$

  Einstein-de Sitter model $k = 0, \lambda = 0$

Expanding models

Einstein's steady–state model (~1931): λ = energy of the vacuum?

"My greatest blunder"

- **Einstein's description of cosmic constant term**

  Reported by George Gamow

- **Controversy**

  Queried by Straumann, Livio

  Not in Einstein's papers or other reports

Georges Gamow

- **Our findings**

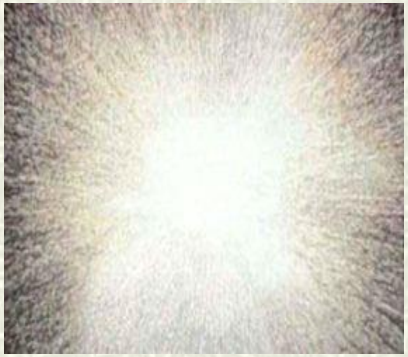
  Consistent with actions

  Einstein's remark reported by Gamow, Alpher, Wheeler

- **Meaning of remark**

  Failure to spot instability of static solution

  Failure to predict expanding universe



Coda: The Einstein World today

- **The question of origins**

  BB model ≠ a theory of origins

  The singularity problem

  The quantum gravity problem

- **<u>The cyclic universe</u>**

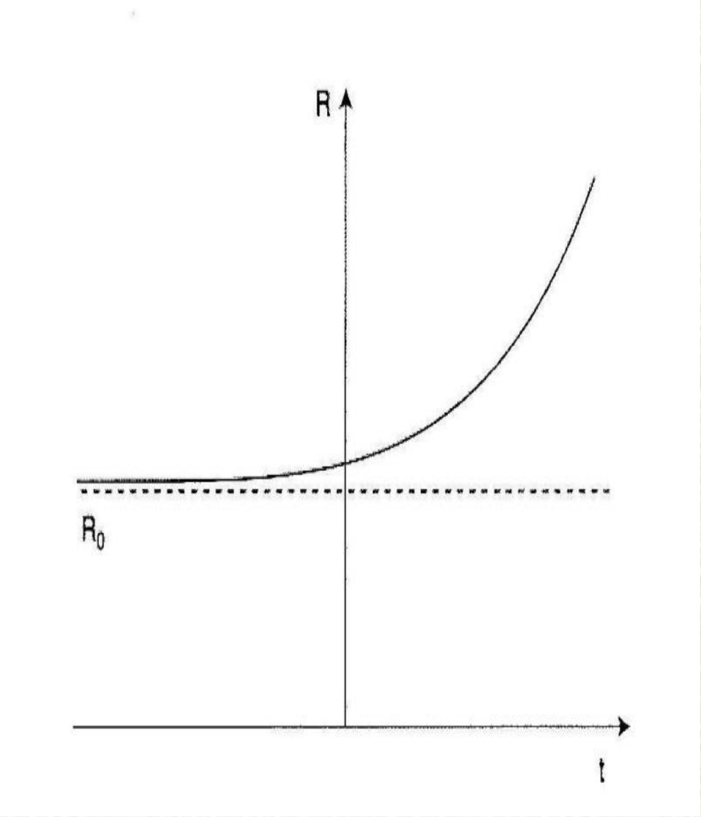
  From BC to BB

- **<u>The emergent universe</u>**

  Inflating from a static Einstein World

- **On the stability of the Einstein World**

  Advanced GR: LQG, DGR, B-D, f(R), f (R,T)



Relevance of past theories in modern science

Einstein's 1917 Static Model of the Universe: A Centennial Review

Cormac O'Raifeartaigh, Michael O'Keeffe, Werner Nahm, Simon Mitton

(Submitted on 25 Jan 2017 (v1), last revised 10 May 2017 (this version, v2))

We present a historical review of Einstein's 1917 paper 'Cosmological Considerations in the General Theory of Relativity' to mark the centenary of a key work that set the foundations of modern cosmology. We find that the paper followed as a natural next step after Einstein's development of the general theory of relativity and that the work offers many insights into his thoughts on relativity, astronomy and cosmology. Our review includes a description of the observational and theoretical background to the paper; a paragraph-by-paragraph guided tour of the work; a discussion of Einstein's views of issues such as the relativity of inertia, the curvature of space and the cosmological constant. Particular attention is paid to little-known aspects of the paper such as Einstein's failure to test his model against observation, his failure to consider the stability of the model and a mathematical oversight concerning his interpretation of the role of the cosmological constant. We recall the response of theorists and astronomers to Einstein's cosmology in the context of the alternate models of the universe proposed by Willem de Sitter, Alexander Friedman and Georges Lemaitre. Finally, we describe the relevance of the Einstein World in today's 'emergent' cosmologies.

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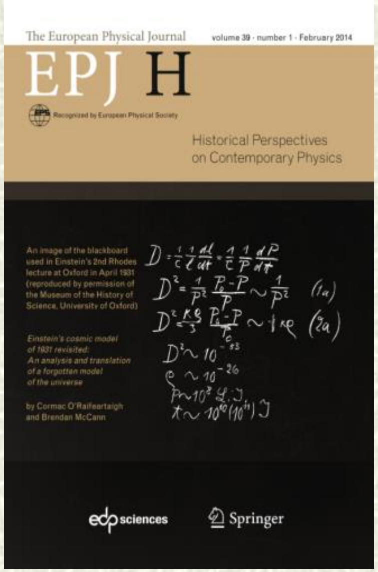
Subjects: **History and Philosophy of Physics (physics.hist-ph)**; Cosmology and Nongalactic Astrophysics (astro-ph.CO)
Cite as: arXiv:1701.07261 **[physics.hist-ph]**
(or arXiv:1701.07261v2 **[physics.hist-ph]** for this version)

The Friedman-Einstein model



- **First translation into English**

  O'Raifeartaigh and McCann 2014

- **<u>Not a cyclic model</u>**

  "Model fails at P = 0 "

  Contrary to what is usually stated

  $$P \sim \frac{1}{D} \qquad D^2 \sim \kappa\rho$$

- **Anomalies in calculations of radius and density**

  Einstein: $P \sim 10^8$ lyr, $\rho \sim 10^{-26}$ g/cm^3, $t \sim 10^{10}$ yr

  We get: $P \sim 10^9$ lyr, $\rho \sim 10^{-28}$ g/cm^3, $t \sim 10^9$ yr

- **Source of error?**

  Oxford blackboard: $D^2 \sim 10^{-53}$ cm^{-2} *should be* 10^{-55} cm^{-2}

  Time miscalculation $t \sim 10^{10}$ yr (*should be* 10^9 yr)

  Non-trivial error: misses conflict with radioactivity